Class 6 - Circles

Review:

$\rightarrow (a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

$(x+1)^2 = x^2 + 2x + 1$

$(x+5)^2 = x^2 + 10x + 25$

$\rightarrow (a-b)^2 = a^2 - 2ab + b^2$

$(x-3)^2 = x^2 - 2 \cdot x \cdot 3 + 3^2 = x^2 - 6x + 9$

$(x-6)^2 = x^2 - 12x + 36$

$(x-7)^2 = x^2 - 14x + 49$

$(x-8)^2 = x^2 - 16x + 64$

$y^2 + 8y + 16 = y^2 + 4y + 4y + 16$

$= y(y+4) + 4(y+4)$

$= (y+4)(y+4)$

$= (y+4)^2$

$(y+4)^2$

↑

$y^2 + 8y + 15 = y^2 + 8y + 15 + 1 - 1$

$= y^2 + 8y + 16 - 1$

$= (y+4)^2 - 1$

If we have $y^2 + 8y + 15 = 0$, we can solve it:

$y^2 + 8y + 15 = 0$

$(y+4)^2 - 1 = 0$

$+1 \quad +1$

$\sqrt{(y+4)^2} = \sqrt{1}$

$y + 4 = \pm 1 \Rightarrow$ $\boxed{y = 4 \pm 1}$

$-4 \quad -4$

Ex1 : Solve $x^2+4x+3=0$.

$x^2+4x+3+1=0+1$

$\sqrt{(x+2)^2}=\sqrt{1}$

$x+2=\pm 1$

$-2 \quad -2$

$\boxed{x=-2\pm 1}$ or $x=-3, x=-1$

1st : Complete the square. $(x+2)^2$

$(x+2)^2=x^2+4x+4$

2nd : solve.

Ex2 : $x^2-10x=6$

1st : What are we looking for? (on the lef-hand side) $(x-5)^2=x^2-10x+25$

2nd : What to add on both sides?

$x^2-10x+25=6+25 \Rightarrow (x-5)^2=31$

3rd : solve.

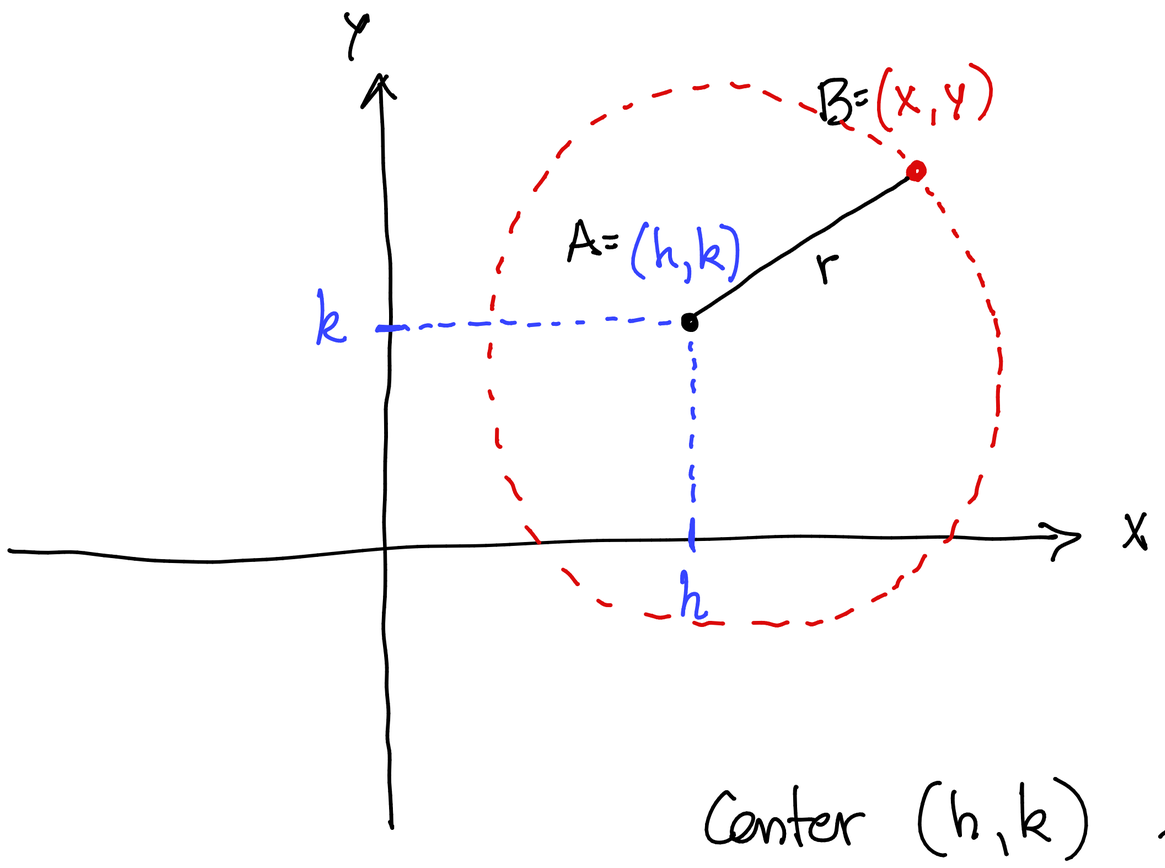
$\sqrt{(x-5)^2}=\sqrt{31} \Rightarrow x-5=\pm\sqrt{31} \Rightarrow x=5\pm\sqrt{31}$

Ex3 : $y^2+8y=36$. Solve for y using the square method.

1st : Looking for $(y+4)^2=y^2+8y+4^2=y^2+8y+16$

2nd : Add 16. $y^2+8y+16=36+16$

3rd : solve. $\sqrt{(y+4)^2}=\sqrt{52} \Rightarrow y+4=\pm\sqrt{52} \Rightarrow y=-4\pm 2\sqrt{13}$.



$$(r)^2 = d(A,B) = \left(\sqrt{(x-h)^2+(y-k)^2}\right)^2$$

Circle with radius r & center (h,k):

Center (h,k) → $(x-h)^2+(y-k)^2=r^2$ **Standard Form**

Ex 1: Circle with center $(1,2)$ (h k) and radius 5.

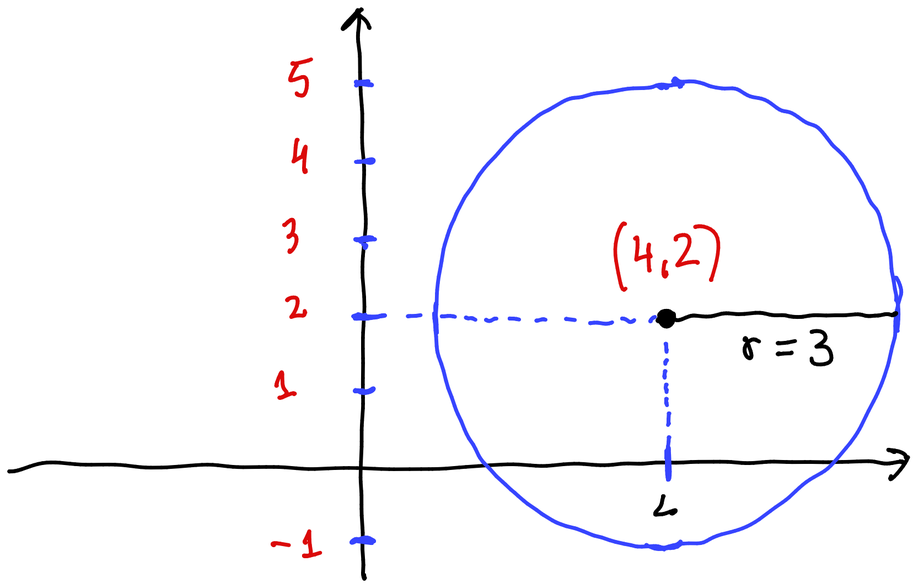
$(x-1)^2+(y-2)^2=5^2$ or $(x-1)^2+(y-2)=25$.

Ex 2: Write in std form a circle with radius 4 and center $(-6,2)$. (h k)

$(x-h)^2+(y-k)^2=r^2 \Rightarrow (x-(-6))^2+(y-2)^2=4^2$

$\Rightarrow (x+6)^2+(y-2)^2=16$

Ex 3: write the equation of the following circle in std form:



$(x-4)^2+(y-2)^2=3^2=9$ → std form

$(x^2-8x+16)+(y^2-4y+4)=9$

$-9 \quad -9$

$x^2+y^2-8x-4y+11=0$ → general form

General form: $Ax^2 + By^2 + Cx + Dy + E = 0$

Ex 4: Find the center and radius of the circle represented by the equation $x^2 + y^2 - 12x - 16y + 75 = 0$.

$x^2 - 12x + 36 \quad y^2 - 16y + 64 + 75 = 0 + 36 + 64$

$(x-6)^2 \qquad (y-8)^2$

$(x-6)^2 + (y-8)^2 + 75 = 100 \Rightarrow (x-6)^2 + (y-8)^2 = 25 = 5^2$

$\quad -75 \quad -75$

Center: $(6, 8)$, Radius: 5

Ex 5: Find the center and radius of the circle represented by the equation $3x^2 + 3y^2 + 6y - 105 = 0$

$\frac{3x^2 + 3y^2 + 6y - 105}{3} = \frac{0}{3} \Rightarrow x^2 + y^2 + 2y - 35 = 0$

$x^2 + 0y + 0 + y^2 + 2y + 1 - 35 = 0 + 0 + 1$

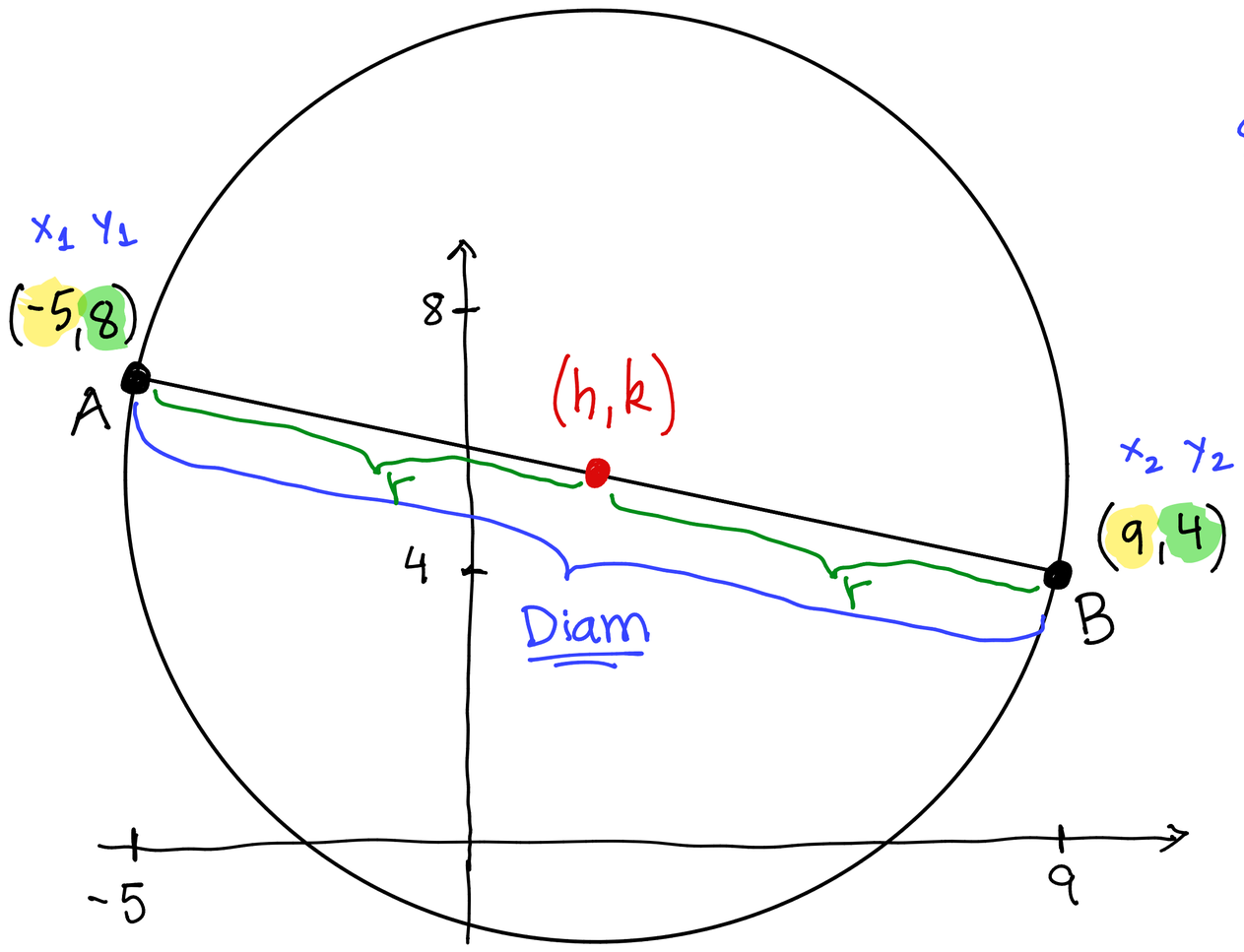
$\updownarrow$

$(x-0)^2 + (y+1)^2 - 35 = 1 \Rightarrow (x-0)^2 + (y+1)^2 = 6^2$

$\quad +35 \quad +35$

Center: $(0, -1)$, radius: 6

Exercise: Find std form of the equation of the circle with endpoints of a diameter at the points (9,4) & (-5,8).



std form: $(x-h)^2+(y-k)^2=r^2$

center: (h,k) — radius: r

$D=2r \iff \frac{D}{2}=r$

$d(A,B)=\sqrt{(9-(-5))^2+(4-8)^2}$

$=\sqrt{14^2+(-4)^2}$

$=\sqrt{196+16}$

$=\sqrt{212}=\sqrt{4\cdot 53}$

$=\sqrt{4}\cdot\sqrt{53}=2\sqrt{53}$

$\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$

$D=d(A,B)=2\sqrt{53}$

$r=\frac{2\sqrt{53}}{2} \Rightarrow \boxed{r=\sqrt{53}}$

$A=(x_1,y_1)$ & $B=(x_2,y_2) \Rightarrow M=(h,k)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$A=(-5,8)$ & $B=(9,4)$

$(h,k)=\left(\frac{-5+9}{2},\frac{8+4}{2}\right)$

$=\left(\frac{4}{2},\frac{12}{2}\right)=(2,6)$

Center = (2,6)

Radius = $\sqrt{53}$

STD FORM

$(x-2)^2+(y-6)^2=53$